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On Possibility to Increase the TMCI Threshold by RF Quadrupole

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Abstract

Transverse mode coupling instability is one of the major limitations of a single bunch current in storage rings. Up to now it appeared in large electron-positron machines, while its presence in proton colliders is under question.

This paper is devoted to a theoretical analysis of the effect of longitudinal variation of the betatron tune (induced by RF quad) on transverse mode coupling instability threshold. It is shown, that it is possible to significantly enhance the threshold, introducing the difference of betatron tunes for the head and the tail of a bunch (due to RF quad) comparable with the synchrotron tune.

1 Introduction

Recently [1] it was discovered by simulations, that incoherent tuneshift can increase the threshold current for a wall impedance. Probably the reason of this is in dependence of betatron tune on longitudinal coordinate. Here we briefly present calculations for the transverse mode coupling instability (TMCI) threshold; one can find general details and features of this instability in [2, 3].

For the purpose of this article one transverse and longitudinal degrees of freedom are considered. The definitions of Wake-functions correspond to [3];

all the results are obtained in general form for distributed impedances. After that, particular case of constant Wake-function [1] was investigated.

The action of RF quad is expressed in dependence of the betatron tune on longitudinal coordinate:

$$\nu_b = \nu_0 + s \cdot g; \tag{1}$$

where ν_0 is the initial betatron frequency, s is the longitudinal coordinate of a particle from the center of the bunch, g is the gradient of betatron frequency proportional to the strength of RF quadrupole [2].

It will be shown further, that for such gradients g, which produce the difference of the betatron frequencies for head and tail particles comparable with synchrotron tune, the TMCI threshold enchances in factor of 2 and it grows with an increasing the gradient g.

The physical reason for such a behavior of this instability versus the longitudinal gradient of betatron tunes is as follows: during a half of synchrotron oscillation forward particles produce the changing of betatron phase and amplitude of backward ones; when the change of betatron phase over the synchrotron period is of the order of unity, this instability occurs. The betatron frequencies are usually the same for the head and the tail of a bunch, so the particles are always in resonance. When the longitudinal gradient of the betatron frequencies exists, the particles have different betatron frequencies. The higher is this gradient, the smaller is the changing of the betatron phase due to their interaction through Wake, because the particles become far from resonance. It is evident, that the effect must increase with increasing of the gradient g. In the next section the simple model of a "hollow beam" will be presented; then all the calculations will be carried out for a general case.

2 "Hollow-beam" model

It's usually convenient to study some simple model in order to understand general properties of eigenvalues and their dependence on parameters. At first we use model of the bunch, which consists of particles with one synchrotron amplitude. Let A and ψ be the amplitude and slow phase of betatron oscillations of the test particle. The averaged equation for them reads

[1] it corresponds, for example, to the Wake-function of a strip line

[2] the wavelength of RF oscillations is assumed to be much larger, than the bunch length

(the chromaticity and the longitudinal gradient of betatron frequencies are equal to zero):

$$\frac{dAe^{i\psi}}{d\tau} = \frac{\omega_b \beta^{3/2}}{i\gamma mc^2} \int_0^T F e^{-i\omega_b \tau} \frac{d\tau}{T} = \overline{F}, \tag{2}$$

where the integration time T must be larger than the betatron oscillation period; ω_b, β are the betatron frequency and β-function, $\omega_b d\tau = dz/\beta$ and the force F is:

$$F = \frac{e^2}{L} \int_s^\infty W(\Delta s) D(\Delta s) \rho(\Delta s) ds,$$

where Δs is the distance between the forward and the test particle, W is the Wake-function of a vacuum chamber, D and ρ are the average transverse dipole moment and density of the forward particles, L is the circumference of the machine.

After averaging with using relation $\omega_b d\tau = dz/\beta$ the equation for hollow beam (for zero longitudinal gradient of the betatron frequency) reads:

$$\frac{dAe^{i\psi}}{d\tau} = \frac{Ne^2 \beta}{i4\pi\gamma mc^2 T_0} \int_{-|\phi|}^{|\phi|} W(\Delta s) \mathcal{D}(\phi') d\phi', \tag{3}$$

where β-function is supposed to be constant for simplicity, ϕ is the synchrotron phase [3], N is the number of particles,$\Delta s = acos(\phi) - acos(\phi')$, T_0 is the revolution frequency and $\mathcal{D} = \frac{D}{\sqrt{\beta}}$ is the average normalized dipole moment $\mathcal{D} = Ae^{i\psi}$.

For nonzero longitudinal gradient of betatron frequencies the phase ψ consists of two parts (see 1):

$$\psi = \Psi + g \cdot \int^\tau s d\tau = \Psi + \frac{g \cdot \lambda E_0}{U} \delta,$$

where τ is time, Ψ is the slow part of the betatron phase, λ and U is the wavelength and amplitude of the RF system, E_0 is the energy of particles, $\delta = \Delta E/E_0$. It is convenient to use a new variable $Ae^{i\Psi}$ because of the fact, that A and Ψ here are influenced only by a collective force. One can easily obtain the equation for this variable:

$$\frac{dAe^{i\Psi}}{d\tau} = e^{-i\frac{g \cdot \lambda E_0}{U}\delta} \frac{Ne^2 \beta}{i4\pi\gamma mc^2 T_0} \int_{-|\phi|}^{|\phi|} W(\Delta s) \mathcal{D}(\phi') d\phi'. \tag{4}$$

[3] the module ϕ in this formulas was written due to the symmetry of the collective force on the synchrotron phase

Taking into account, that $\mathcal{D} = Ae^{i\Psi + i\frac{g\cdot\lambda E_0}{U}\delta}$, and after rewriting the total derivative on time via partial derivatives on time and synchrotron phase, previous equation converts into:

$$\frac{\partial d}{\partial \tau} + \frac{\partial d}{\partial \phi} = \frac{Ne^2\beta}{i4\pi\gamma mc^2 T_0} e^{-iP\sin(\phi)} \int_{-|\phi|}^{|\phi|} W(\Delta s) d(\phi') e^{iP\sin(\phi')} d\phi', \tag{5}$$

where $d = Ae^{i\Psi}$ and parameter $P = \frac{g\cdot\lambda E_0}{U} a_e$ is just a half of differences of betatron phases due to RF quadrupole for this particular synchrotron amplitude.

Then for finding eigenfrequencies it is convenient to present d in the form of infinite sum of harmonics of synchrotron frequency multiplied by exponent function of time :

$$d = e^{i\alpha\tau} \sum_{n=-\infty}^{+\infty} d_n e^{-in\phi},$$

where α is some eigenfrequency. After putting it in the previous equation, multiplying it by $e^{in\phi}$, integrating equation over synchrotron phase from $-\pi$ to π and rearranging the terms, one can obtain:

$$d_n(\alpha - n\omega_s) = -K \sum_{m=-\infty}^{+\infty} d_m K_{nm}, \tag{6}$$

where ω_s is the synchrotron frequency, $K = \frac{Ne^2\beta}{2\pi^2\gamma mc^2 T_0}$ and
$K_{nm} = \int_0^\pi cos(n\phi - P\sin(\phi))d\phi \int_0^\phi W(\Delta s) cos(m\phi' - P\sin(\phi'))d\phi'$.

Usually, the sum is truncated to a finite number of lower modes. In the case of only two lower modes $n = 0, -1$ the matrix for finding eigenfrequencies is:

$$\begin{array}{cc} \alpha + K_{00}\cdot K & K_{01}\cdot K \\ K_{10}\cdot K & \alpha + \omega_s + K_{-1-1}\cdot K. \end{array} \tag{7}$$

Putting the determinant of this matrix equal to zero, one can obtain an equation for α. For zero P $K_{01} = -K_{10}$ and an imaginary part of eigenfrequencies appears for some threshold current (see, for example, [2]). It's evident, that when K_{01}, K_{10} have the same sign, this quadratic equation never gives imaginary solutions, so the TMCI instability in this model disappears. Fig 1 shows K_{01}, K_{10} versus parameter P for constant Wake $W = 1$. For $P \simeq 0.77$ the

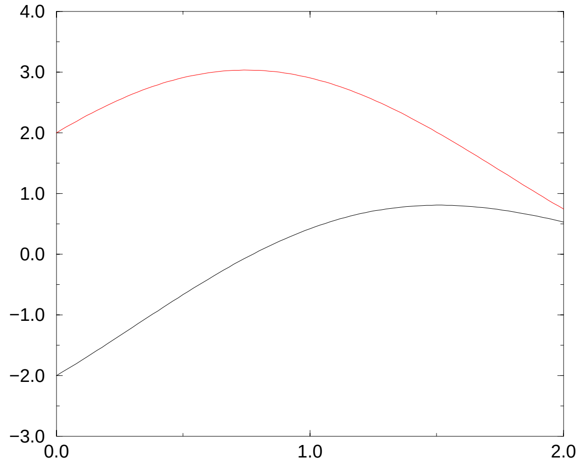

Figure 1: The coefficients K_{01} (upper) and K_{10} (lower) versus the parameter P.

coupling terms become of the same sign and TMCI disappears for such a simple model. For another sign of P the figure of K_{01}, K_{10} versus P can be obtained by reflection of Fig. 1 over the zero x axis, so the threshold depends on the modulus P [4].

3 General investigation

The equation for gaussian distribution in synchrotron phase space can be obtained in the same manner:

$$\frac{\partial d(a,\phi)}{\partial \tau} + \frac{\partial d(a,\phi)}{\partial \phi} = \frac{Ne^2\beta}{i4\pi\gamma mc^2 T_0 \sigma^2} e^{-iPa/\sigma \cdot \sin(\phi)}$$

$$\int_0^\infty a' da' \exp(-(a')^2/2\sigma^2) \int_{-F(a,a',|\phi|)}^{F(a,a',|\phi|)} W(\Delta s) d(a',\phi') e^{iPa'/\sigma \cdot \sin(\phi')} d\phi', \qquad (8)$$

where a is the synchrotron amplitude, σ is the longitudinal R.M.S. size, P is the half of the maximum betatron phase difference for particles with the

[4] the same is valid for all the next results also

positive and negative energy offsets for $a = \sigma$, $\Delta s = acos(\phi) - a'cos(\phi')$ and $F(a, a', \phi)$ is determined by:

$$F(a, a', \phi) = acos(a/a'cos(\phi)), if|a/a'cos(\phi)| < 1;$$

$$F(a, a', \phi) = \pi \cdot sign(cos(\phi)), otherwise.$$

For practical calculations of the eigenvalues of this problem, the bunch is divided into rings with the fixed amplitudes in the synchrotron phase space. So the dimension of this system increases in factor equal to the number of the rings in comparison with the "hollow beam" model. The coupling coefficient of some mode with azimuthal number n and amplitude a with some mode with azimuthal number m and amplitude b is:

$$K_{nmab} = R \int_0^{\pi} cos(n\phi - P\sin(\phi)a/\sigma)d\phi$$

$$\int_0^{F(a,b,\phi)} W(\Delta s)cos(m\phi' - P\sin(\phi')b/\sigma)d\phi',$$

where $R = \frac{Ne^2\beta b\exp(-b^2/2\sigma^2)}{2\pi^2\gamma mc^2T_0\sigma^2}$.

The linear equation for eigenmodes is:

$$d_n(a_i)(\alpha - n\omega_s) = -\sum_j \sum_{m=-\infty}^{+\infty} d_m(a_j)K_{nma_ib_j}, \quad (9)$$

where α is the eigenfrequency, i, j mean the indices of the rings in synchrotron phase space. The smaller are the rings, the closer to their actual values are the eigenvalues. They can be found from the equation, which can be obtained after putting the determinant the above matrix equal to zero.

Further, the results of eigenvalues calculation are shown. The bunch is divided into 5 radial rings and each ring is presented by 5 azimuthal modes, so it is possible to see the behavior of the first 25 modes, which gives usually few percent deviation for the threshold from its actual value.

In Fig.(2) one can see the eigentunes for the constant Wake-function W versus parameter $X = \frac{Ne^2\beta W}{2\pi^2\gamma mc^2T_0}$, which is proportional to the number of particles per bunch. In this figure the betatron tune is shifted into zero . The synchrotron tune here is equal to unity and parameter P is equal to zero. All the real parts of frequencies start from a zero current; the imaginary parts

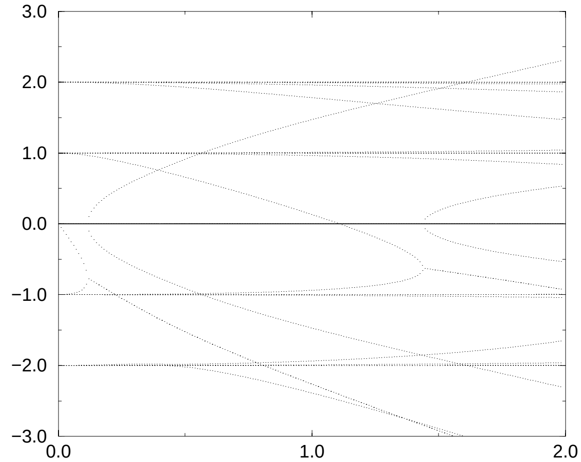

Figure 2: The eigenfrequencies of transverse oscillations versus parameter $X = \frac{Ne^2\beta W}{2\pi^2\gamma mc^2 T_0}$. The parameter $P = 0$

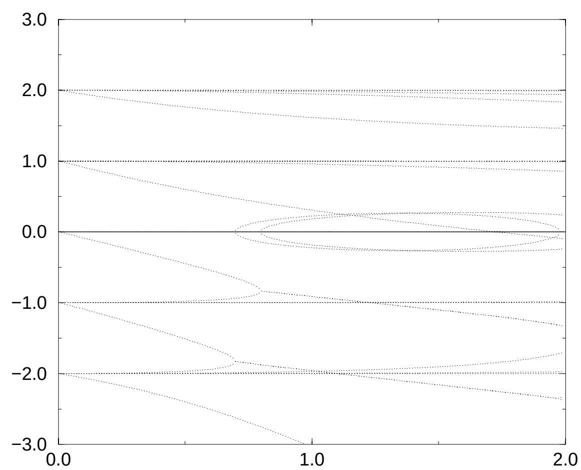

Figure 3: The eigenfrequencies of transverse oscillations versus parameter $X = \frac{Ne^2\beta W}{2\pi^2\gamma mc^2 T_0}$. The parameter $P = 2$.

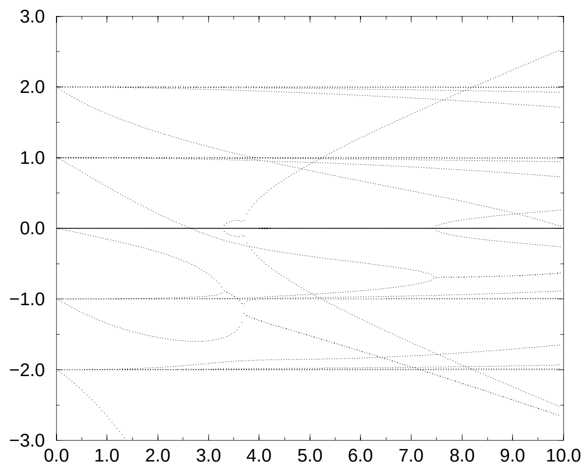

Figure 4: The eigenfrequencies of transverse oscillations versus parameter $X = \frac{Ne^2\beta W}{2\pi^2\gamma mc^2 T_0}$. The parameter $P = 5$.

appear near the zero axis after merging of some modes. Near betatron tune there is a bunch of the "radial" eigentunes with zero "azimuthal" number. If briefly, all these modes for small current have zero oscillations of dipole moment over an angle in synchrotron phase space, and they differ in dependence of the dipole moment on the synchrotron amplitude ("radius" in synchrotron phase space). For each integer number there are higher "azimuthal" modes, whose tunes differ from betatron tune in this particular integer number of synchrotron tunes for small current. This number means the number of modulation of eigenmodes over the angle in synchrotron phase space. As in the case of "zero" azimuthal modes, there are a lot of "radial" modes for every azimuthal number. The first merging of some "zero" radial mode and some "-1" radial mode occurs for $X = .12$. The next merging occurs for 5 times larger current ("-1" radial and "+1" radial modes). In both cases a pair of the modes with equal real parts of the tune and with opposite imaginary parts of the tune appears; this, evidently, means instability of the bunch.

Figures 3,4 show the eigenvalues for $P = 2$ and $P = 5$ consequently. Finally the factor of increasing the TMCI threshold is shown in Fig. 5. One

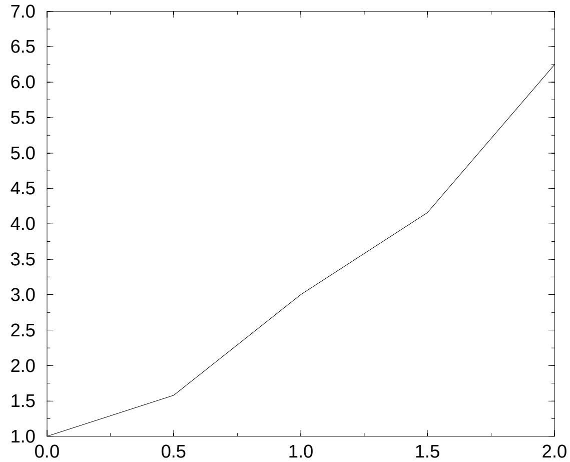

Figure 5: The factor of increasing the TMCI threshold versus parameter P

can see, that the RF quad easily can shift the threshold in factor five and more.

4 Conclusion

It is shown, that the longitudinal gradient of betatron frequency can effectively increase the TMCI threshold. The similar ideas of using the shift of transverse frequency along the bunch was earlier proposed for linacs [5].

Rough estimations for the VLHC [4] can be made for the length of such a quadrupole. Let's take the maximum electric field equal to 50 MV/m, aperture and wavelength of RF equal to 10 cm (the designed bunch length is approximately the same), beta-function equal to 600 meters, the synchrotron tune equal to .01 and the injection energy is 3 TeV. The result of the calculations is, that one needs 1.25 meters of quadrupole to produce betatron tuneshift of about one synchrotron tune for particles with 1 R.M.S. longitudinal offset. Probably it is possible to combine RF quadrupole with the basic RF system. In this case the same RF generators can be used. So it looks like this method of increasing the TMCI threshold may be simple and reliable.

Acknowledgments

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References

[1] M.Blaskiewicz, "Effect of incoherent space charge forces on the fast head-tail instability", presented at 1997 PAC, Vancouver, Canada.

[2] V.V.Danilov and E.A.Perevedentsev, "Feedback system for elimination of the transverse mode coupling instability", NIM A, 391 (1997) p.77-92.

[3] A.W.Chao, "Physics of collective beam instabilities in high energy accelerators", John Wiley & Sons, Inc., 1993

[4] A low field approach to a Very Large Hadron Collider, Summer Study on new directions for High-Energy Physics Snowmass '96.

[5] V.E.Balakin, A.V.Novokhatsky and V.P.Smirnov, "VLEPP: transverse beam dynamics",Proc. of the 12 International Conference On High-Energy Accelerators, Fermilab, 1983